A Locate and classify the singular points

(if exist):

①  $x(x-1)^3 \frac{2^3y}{4x^2} - 2(x-1)^2y^1 + 5y = 0$ It is normalized formed is  $y'' - \frac{2}{x(x-1)}y' + \frac{5}{x(x-1)^3}y' = 0$ 

Here,  $P(x) = -\frac{2}{x(x-1)}$ , is not analytic at x = 0 and x = 1

 $O(x) = \frac{5}{2(x-1)^3}$ , is not analytic at x=0 and x=1

:. a=0,1 are the ringular points.

Now,  $(2-0) \cdot p(x) = -\frac{2x}{2(x-1)} = -\frac{2}{x-1}$ , is analytic at x = 0

 $(2x-0)^2 8(2x) = \frac{52}{(2x-1)^3}$ , is analytic at 2x=0

: x = 0 is a regular singular point.

Again,  $(a-i)P(a) = -\frac{2}{2}$ , its analytic at a=1

 $((x-1)^2 O(x) = \frac{5}{\pi(x-1)}$ , its not analytic at x = 1.

: x=1 is an irregular singular point.

(x2-3x) y"+(x+1)y'-2y=0 It's normalised form its  $+\frac{2x+1}{2(2x-3)}y^{1}-\frac{2}{2(2x-3)}y=0$  $p(x) = \frac{x+1}{x(x-3)}$ , is not analytic at. x=0 and x=3 $O(2) = -\frac{2}{2(2-3)}$ , is not analytic at :.  $\alpha = 0$ , 3 we the singular points.  $(\alpha - 0)P(\alpha) = \frac{\alpha + 1}{\alpha - 3}$ , is analytic at  $\alpha = 0$ 

- 22 , is analytic at n=0

- x = 0 is a regular singular point.

- 2+1 , is analytic at n=3

 $(2-3)^2 O(2) = -\frac{2(2-3)}{2}$ , is analytic at 2=3

is 21=3 is a regular singular point.

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2. 
$$3^2(3+1)^2\eta'' + 2(3^2-1)\eta' - 3\eta = 0$$
  
It is normalised form is

$$3'' + \frac{2(x^2-1)}{x^2(x+1)^2} 3' - \frac{3}{x^2(x+1)^2} 3 = 0$$

Here, 
$$P(x) = \frac{2(x^2-1)}{x^2(x+1)^2}$$
, is not analytic at  $x=0,-1$ 

$$8(x) = -\frac{3}{3^2(3+1)^2}$$
, is not analytic at  $x = 0$ ,  $-1$ 

Now,

$$(2-0)P(x) = \frac{2(x^2-1)}{2(x+1)^2}$$
, is not analytic at  $x=0$ 

$$(x-0)^2B(x) = -\frac{3}{(x+0)^2}$$
, is analytic at  $x=0$ 

Again, 
$$(\chi+1) P(\chi) = \frac{2(\chi^2-1)}{\chi^2(\chi+1)}$$
, is an analytic  $= \frac{2(\chi+1)}{\chi^2}$  at  $\chi = -1$ 

$$(2+1)^{2}S(x) = -\frac{3}{x^{2}}$$
, ivs woralytic at  $x=-1$ 

 $(2^{3}+3^{2})$   $3^{11}+(2^{2}-23)^{3}1-3=0$ Its normalized form is  $y'' + \frac{x(x-2)}{x^2(x+1)}y' - \frac{1}{x^2(x+1)}y = 0$  $\Rightarrow y'' + \frac{(n-2)}{n(n+1)}y' - \frac{1}{n^2(n+1)}y = 0$ Here,  $p(x) = \frac{x-2}{x(x+1)}$ , is not analytic at x=0, -1 $B(x) = -\frac{1}{x^2(x+1)}$ , its not analytic at x = 0, -1íc : x = 0, -1 are the singular points.  $(2-0)P(2) = \frac{2-2}{2(+1)}$ , is avalytic at  $(x-0)^2O(n) = -\frac{1}{n+1}$ , is analytic at n=0-: x = 0 ivs a regular riongular point. e Č Again,  $(\alpha+1)P(\alpha) = \frac{(\alpha-2)}{\alpha}$ , its analytic at  $\alpha=-1$  $(x+1)^2 8(x) = -\frac{x+1}{x^2}$ , is analytic at x=-1 $\therefore x = -1$  is a regular singular point.

 $\frac{5}{2} (x^{4} - 2x^{3} + x^{2})y'' - 4(x-1)y' - 5x^{2}y = 0$   $\frac{5}{2} \text{ possible of form is } x^{4} - 2x^{3} + x^{2}$   $y'' - \frac{4}{x^{2}(x-1)}y' - \frac{5}{(x-1)^{2}}y' = 0 = x^{2}(x^{2} - 2x + 1)$   $= x^{2}(x-1)^{2}$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{4}{x^{2}(x-1)}y' - \frac{5}{(x-1)^{2}}y' = 0$   $= x^{2}(x-1)^{2}$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{4}{(x-1)}y' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{4}{(x-1)}y' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$   $+ \exp(-x^{2} - 2x^{3} + x^{2})y'' - \frac{5}{(x-1)^{2}}y' = 0$ 

is not analytic at n=0,1

 $O(2) = -\frac{5}{(2-1)^2}$ , is not analytic at n=1

x = 0, L are the singular points.

Now,  $(x-0) \cdot P(x) = -\frac{4}{x(x-1)}$ , is not analytic at x=0

 $(2(-0)^2 \cdot O_3(x) = -\frac{5x^2}{(2(-1)^2)}$ , is analytic at

-- x = 0 is an irregular singular point.

Again,  $(x-1) \cdot P(x) = -\frac{4}{x^2}$ , is avalytic at x=1

 $(5(-1)^{2}\cdot 3(2) = -5$ , its wordlytic at everywhere.

· n=1 irs a regular riorgular point.

 $(x^{2}+x^{2})y'' + (x^{2}-2x)y'' + 4y = 0$   $\Rightarrow x^{2}(x+1)y'' + x(x-2)y' + 4y = 0$ It is normalized form in  $y'' + \frac{(x-2)}{x(x+1)} + \frac{4}{x^{2}(x+1)}y = 0$ Here,  $p(x) = \frac{x-2}{x(x+1)}$ , is not analytic at x=0,-1  $\cdot 8(x) = \frac{4}{x^{2}(x+1)}$ , its not analytic at x=0,-1  $\cdot x = 0,-1$  are the singular points.

Now,  $(x-0) \cdot p(x) = \frac{x-2}{x+1}$ , is analytic at x=0.

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Now,  $(x-0) \cdot P(x) = \frac{x-2}{x+1}$ , is analytic at x=0  $(x-0)^2 \cdot S(x) = \frac{4}{x+1}$ , is analytic at x=0 -x = 0 is a regular singular point.

Again,  $(x+1) \cdot P(x) = \frac{x-2}{x}$ , is analytic at x=-1  $(x+1)^2 \cdot O(x) = \frac{4(x+1)}{x^2}$ , is analytic at x=-1 -x = -1 is a regular singular point.

 $(x^{5}+x^{4}-6x^{3})y^{11}-x^{2}y^{1}-2(x-2)y=0$ Itis normalized form is | 25+24-623  $y'' - \frac{1}{x(x-2)(x+3)}y' - \frac{2}{x^3(x+3)}y = 0 = x^3(x^2 + x - 6)$   $= x^3(x-2)(x+3)$ Here,  $P(x) = -\frac{1}{\pi(\pi-2)(\pi+3)}$ , its onet analytic at  $\pi=0, 2, -3$  $O(x) = -\frac{2}{x^3(x+3)}$ , is not analytic at : 2=0,2,-3 are the morgular points.  $(x-0) \cdot P(x) = -\frac{1}{(x-2)(x+3)}$  is analytic at x=0 $(2-0)^2 \otimes (2) = -\frac{2}{2(2+3)}$ , is not analytic : a = 0 is an ir regular sarogular point. Ageira,  $(x-2) \cdot P(x) = -\frac{1}{2(2+3)}$ , is analytic at x=2 $(2-2)^2 B(x) = \frac{2(x-2)^2}{3(3(x+3))}$ , is analytic at x=2is a regular singular point. Aloso,  $(x+3)\cdot P(x) = -\frac{1}{x(x-2)}$ , is analytic at x=-3 $(2+3)^{2}S(2) = -\frac{2(2+3)}{2^{3}}$ , is analytic is = -3 is a regular ratingular point.

find the nature of the point x=0 for the ODE @ (a+1) 2y"+ (sim x)y = 0 It's normalised form is 7"+ Simx y =0  $S(x) = \frac{3}{(x+1)x} = \frac{2}{11} - \frac{2}{12} + \frac{25}{15}$ Here, p(x) = 0 glic = 1 - 12 + 24 -. so, clearly at 21 = 0, both p(21) and B(21) are analytic. => x = 0 is an ordinary point. **=**0 27y" - (corsa) y = 0 Its normalized form is C 7"- CONSX 7 = 0 Here, p(n) = 0 -2  $\mathcal{B}(x) = -\frac{\cos x}{2x}$  $=-\frac{\left(1-\frac{2^{2}}{12}+\frac{24}{14}-\frac{26}{16}+\cdots\right)}{16}$ 7=2 so, clearly at x =0, P(x) is analytic ↞ but B(0) is not analytic. :. 2 = 0 is a ringular point. `C Now, (2-0), p(2) = 0, is analytic at 2=0 (2-0)2. B(2) = - 1 2 cossi, is analytic at 2=0 = 2 = 0 is a regular singular point.

= xy" + 2xy + (tanx) y = 0 It's normalised form is 3"+ 2y + tann y =0 Here, p(su) = 2, is analytic at 2=0  $S(x) = \frac{\tan x}{x}$   $= \frac{x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots}{1 + \frac{x^2}{15} + \frac{2x^4}{15} + \cdots}$ is analytic at x=0 : a = 0 is not ordinary point. = (25-x) 211-xy+ 209 (1+x) y=0 Its normalised foron is  $y'' - \frac{1}{x-1}y' + \frac{\log(1+x)}{2(x-1)}y = 0$ Here,  $p(x) = -\frac{1}{x-1}$ , is analytic at x=0 $S(x) = \frac{\log(1+x)}{x(x-1)} = \frac{2!}{1!} - \frac{x^2}{1!^2} + \frac{x^3}{1!^3} - \cdots$  $=\frac{\frac{1}{11}-\frac{2}{12}}{\frac{1}{12}}+\frac{2^{2}}{12}-\frac{1}{12}$ , is wealytic :x=0 is an ordinary point.

e. 
$$x^2y'' + y' + (sinhx)y = 0$$
  
It so normalized form is  
 $y'' + \frac{1}{x^2}y' + \frac{sinhx}{x^2}y = 0$ 

Here,  $p(x) = \frac{1}{2}$ , is not analytic at x=0

$$S(x) = \frac{\sin hx}{x^{2}}$$

$$= \frac{x + \frac{2^{3}}{13} + \frac{2^{5}}{15} + \frac{2^{7}}{17} + \cdots}{x^{2}}$$

$$= \frac{1 + \frac{2^{3}}{13} + \frac{2^{7}}{15} + \frac{2^{7}}{17} + \cdots}{x^{2}}$$

$$= \frac{1 + \frac{2^{3}}{13} + \frac{2^{7}}{15} + \frac{2^{7}}{17} + \cdots}{x^{2}}$$

is not analytic at x=0

: x = 0 îs a singular point.

=0

Now. Gr-o). P(x) = \frac{1}{x}, is not avalytic at n=0.

 $(x-0)^2$  sum B(x) = sinhx, is analytic at <math>x=0

: x = 0 is an irregular singular point.