Class – 4 (ipsita.rajwar@gmail.com)

(5) Third way → Using Matrix Method

$$\left.\begin{aligned}\frac{dx}{dt} &= x+3y \\ \frac{dy}{dt} &= x-y\end{aligned}\right\}$$

This system can be written as

$$\begin{pmatrix} x \\ y \end{pmatrix}' = X' = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$ where $X = \begin{pmatrix} x \\ y \end{pmatrix}$ & Prime denotes derivative

i.e $X' = AX$

The characterised polynomial of A is

$\det(A - \lambda I) = 0.$

i.e $\begin{vmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = 0 \Rightarrow -(1-\lambda)(1+\lambda) - 3 = 0$

$\Rightarrow \lambda^2 - 4 = 0.$

$\Rightarrow \lambda = \pm 2.$

∴ The Eigenvalues are $2, -2$.

for $\lambda = 2$, let us find eigenvector corresponding to it.

$$(A - 2I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} -x + 3y &= 0 \\ \& \quad x - 3y &= 0.\end{aligned}\right\} \Rightarrow x = 3y.$$

[here the coefficient matrix is $\begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix}$ & $\begin{vmatrix} -1 & 3 \\ 1 & -3 \end{vmatrix} = 0$, so it has infinite solⁿ]

∴ The eigenvector is $\begin{pmatrix} x \\ \frac{1}{3}x \end{pmatrix} = x\begin{pmatrix} 1 \\ 1/3 \end{pmatrix}$

for $\lambda = -2$, the eigenvector is

$$(A + 2I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{l} 3x + 3y = 0 \\ \& \quad x + y = 0 \end{array}\right\} \Rightarrow x = -y$$

$\therefore$ The eigenvector is

$$\begin{pmatrix} x \\ -x \end{pmatrix} = x\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

[here also $\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} = 0 \Rightarrow$ infinite sol^n]

As these ~~eigenvectors are~~ eigenvalues are real & distinct ($\lambda = 2, -2$) we have the general sol^n for X as

$$X = c_1\begin{pmatrix} 1 \\ 1/3 \end{pmatrix} e^{2t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 e^{2t} + c_2 e^{-2t} \\ \frac{1}{3} c_1 e^{2t} - c_2 e^{-2t} \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{l} x = c_1 e^{2t} + c_2 e^{-2t} \\ y = \frac{1}{3} c_1 e^{2t} - c_2 e^{-2t} \end{array}\right\} \quad \underline{\text{Ans.}}$$

Depending on the nature of eigenvalues λ_i's we have

i) All eigenvalues are real

- all are different from each other
- Some are repeated.

ii) Some are complex conjugate

If the eigenvalue λ is a repeated root of the characteristic equation, but the system has only one non zero solution v_1 (upto scalar multiple), then the eigenvalue is said to be incomplete or defective & $x_1 = e^{\lambda_1 t} v_1$ is the unique normal mode. We need another independent solⁿ. & this second independent solⁿ is given by

$$x_2 = e^{\lambda t}(t v_1 + v_2) \text{ where } v_2 \text{ is any}$$

vector satisfying $(A - \lambda I) v_2 = v_1$.

Exercise: Find general solⁿ using different methods.

1) $X' = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix} X$

2) $X' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} X$

3) $X' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} X$

4) $\frac{dx}{dt} = 4x - 3y$

$\frac{dy}{dt} = 8x - 6y$

5) $X' = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} X$

6) $X' = \begin{pmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{pmatrix} X$

7) $X' = \begin{pmatrix} \alpha & 2 \\ -2 & 0 \end{pmatrix} X$, ($\alpha$ being a parameter.)

8) $X' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

9) $X' = \begin{pmatrix} -1 & 3/2 \\ -1/6 & -2 \end{pmatrix} X$, $X(2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

* For any doubt or suggestion you may contact with me through mail →
ipsita.rajwar@mail.com

Assignment-1

(22)

(1) Transform the given equation into a system of first order equations.

i) $u''+2u=0,\quad u(0)=0,\ u'(0)=2.$ — (*).

Sol^n:

Let $x(t)=u(t)$
$y(t)=u'(t)$ } define two new functions x, y. — (1)

$\therefore\ x'(t)=u'(t)=y(t).$ (using (1))

& $y'(t)=u''(t)=-2u=-2x(t)$
(from (*)) (using (1))

initial condition $x(0)=u(0)=0$
$y(0)=u'(0)=2$

Hence we have the system of equations

$$\boxed{\begin{array}{ll} x'=y & \text{with } x(0)=0 \\ y'=-2x & \quad\quad\ \ y(0)=2 \end{array}}$$

Ans.

---

ii) $u''+u'+1.25u=3\cos t,\ u(0)=2,\ u'(0)=3.$

Sol^n: Define two new ~~variables~~ functions x, y as

$x(t)=u(t)$
$y(t)=u'(t)$ } — (1)

$u''+u'+1.25u=3\cos t$ $\therefore$
$u''=-u'-1.25u+3\cos t$

$\therefore\ x'(t)=u'(t)=y(t)$ — (2)

& $y'(t)=u''(t)=-u'-1.25u+3\cos t$

$\therefore\ y'=-y-1.25x+3\cos t$

I.V

$$x(0) = u(0) = 2$$
$$y(0) = u'(0) = 3$$

∴ The required system of equation,

$$\begin{cases} x' = y \\ y' = -y - 1.25x + 3\cos t \end{cases}, \quad \begin{matrix} x(0) = 2 \\ y(0) = 3. \end{matrix}$$

Ans

iii) $u^{(4)} - u = 0.$

Soln:

Here we need four new functions,

Let
$$\left.\begin{aligned} x(t) &= u(t) \\ y(t) &= u'(t) \\ w(t) &= u''(t) \\ z(t) &= u'''(t) \end{aligned}\right\} \text{(I)}$$

Then
$$\left.\begin{aligned} x'(t) &= u'(t) = y(t) \\ y'(t) &= u''(t) = w(t) \\ w'(t) &= u'''(t) = z(t) \\ z'(t) &= u^{(4)}(t) = u = x \end{aligned}\right\}$$
→ from (I)

$$u^4 - u = 0$$
$$u^4 = u$$

∴ required system
$$\left.\begin{aligned} x'(t) &= y \\ y' &= w \\ w' &= z \\ z' &= x \end{aligned}\right\}$$
Ans.

(24)

(2) Transform the given system into a single equation of higher order.

(i) $x_1' = x_1 - 2x_2$
$x_2' = 3x_1 - 4x_2$

Solⁿ:

$$x_1' = x_1 - 2x_2 \quad \text{--- (1)}$$
$$x_2' = 3x_1 - 4x_2 \quad \text{--- (2)}$$

from (1), we get $x_2 = \frac{1}{2}[x_1 - x_1']$

$\Rightarrow x_2' = \frac{1}{2}[x_1' - x_1'']$

Putting these values we have

$$\frac{1}{2}[x_1' - x_1''] = 3x_1 - 4 \cdot \frac{1}{2}(x_1 - x_1')$$

$$\Rightarrow x_1' - x_1'' = 6x_1 - 4x_1 + 4x_1'$$

$$\Rightarrow x_1'' + 3x_1' + 2x_1 = 0.$$

we can rename x_1 as u & hence this system converted to a single equation of 2nd order as

$$\boxed{u'' + 3u' + 2u = 0}$$

(ii) $x_1' = 2x_2$
$x_2' = -2x_1$

Solⁿ. $x_1' = 2x_2 \Rightarrow x_2 = \frac{x_1'}{2}$ & hence $x_2' = \frac{x_1''}{2}$

& $x_2' = -2x_1$

$$\Rightarrow \frac{x_1''}{2} = -2x_1$$

$$\Rightarrow x_1'' + 4x_1 = 0$$

i.e $u'' + 4u = 0$ is the required single eqⁿ for the system of equation.

(3) Use operator method to solve the ~~given~~ linear systems.

(i) $\frac{dx}{dt} + \frac{dy}{dt} - x - 3y = e^{t}$

$\frac{dx}{dt} + \frac{dy}{dt} + x = e^{3t}$

Soln: using the operator symbol $\frac{d}{dt} \equiv D$, we have

$Dx + Dy - x - 3y = e^{t} \Rightarrow (D-1)x + (D-3)y = e^{t} \quad \dots (i)$

$Dx + Dy + x = e^{3t} \Rightarrow (D+1)x + Dy = e^{3t} \quad \dots (ii)$

applying the operator D on (i) & applying $(D-3)$ on (ii)

$D(D-1)x + D(D-3)y = D(e^{t})$

$(D-3)(D+1)x + (D-3)Dy = (D-3)(e^{3t})$

Substract $(\cancel{D^2} - D - \cancel{D^2} + 2D + 3)x = D(e^{t}) - (D-3)(e^{3t})$

$\Rightarrow (D+3)x = e^{t} - 3e^{3t} + 3e^{3t} = e^{t}$

$\therefore \frac{dx}{dt} + 3x = e^{t}$ (1st order linear diff'al eqn)

$I.F = e^{\int 3dt} = e^{3t}$

$\therefore x = e^{-3t}\left[\int e^{3t} \cdot e^{t}\, dt + c_1\right] = \frac{e^{t}}{4} + c_1 e^{-3t}$

$\therefore \boxed{x = \frac{e^{t}}{4} + e^{-3t} c_1}$

$\frac{dx}{dt} = \frac{e^{t}}{4} + (-3)e^{-3t} c_1$

Apply operator $(D+1)$ on (i) & operator $(D-1)$ on (ii) (26)

$$(D+1)\cancel{(D-1)}x+(D+1)(D-3)y=(D+1)e^{t}$$
$$(D-1)\cancel{(D+1)}x+(D-1)Dy=(D-1)e^{3t}$$

$$(D^2-2D-3-D^2+D)y=(D+1)e^{t}-(D-1)e^{3t}$$

$$\Rightarrow (-D-3)y = e^{t}+e^{t}-\cancel{3e^{3t}}+\cancel{3e^{3t}}=2e^{t}$$

$$\therefore \quad Dy+3y=-2e^{t}$$

$$I.F = e^{3t}$$

$$\therefore y = e^{-3t}\left[-2\int e^{3t}\cdot e^{t}dt + c_2\right] = -2\frac{e^{t}}{4}+c_2e^{-3t}$$

$$\boxed{y=\frac{-e^{t}}{2}+c_2e^{-3t}}$$

$$\frac{dy}{dt}=-\frac{e^{t}}{2}+(-3)e^{-3t}\cdot c_2$$

using the coefficient matrix

$$\begin{vmatrix} D-1 & D-3 \\ D+1 & D \end{vmatrix} = \cancel{D^2}-\cancel{D}-\cancel{D^2}+3D+\cancel{D}+3 = 3D+3$$

which is 1 degree.

Hence the no. of arbitrary constant should be 1.

& use the eqn (ii) to find the relation between c_1 & c_2.

$$\frac{e^{t}}{4}-3c_1e^{-3t}-\frac{e^{t}}{2}-3c_2e^{-3t}+\frac{e^{t}}{4}+c_1e^{-3t}=e^{3t}$$

$$\Rightarrow -2c_1e^{-3t}-3c_2e^{-3t}=e^{3t}$$

$$\Rightarrow c_2=\frac{1}{3}\left(-2c_1e^{-3t}-e^{3t}\right)e^{3t}=-\frac{2}{3}c_1-\frac{1}{3}e^{6t}$$

$$\therefore y=-\frac{e^{t}}{2}+\left(-\frac{2}{3}c_1-\frac{1}{3}e^{6t}\right)e^{-3t}=-\frac{e^{t}}{2}-\frac{2}{3}c_1e^{-3t}-\frac{e^{3t}}{3}.$$

hence the soln of the system of eqn is (27)

$$\left.\begin{aligned} x &= c_1 e^{-3t} + \frac{e^t}{4} \\ y &= -2c_1 \frac{e^{-3t}}{3} - \frac{e^{3t}}{3} - \frac{e^t}{2} \end{aligned}\right\} \text{Ans.}$$

(4) (i) Determine whether the vector functions are linearly independent or not.

$$x_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t, \quad x_2 = \begin{pmatrix} 2 \\ 6 \end{pmatrix} e^t + \begin{pmatrix} 8 \\ -8 \end{pmatrix} t e^t$$

Soln:

$$x_1 = \begin{pmatrix} e^t \\ -e^t \end{pmatrix}, \quad x_2 = \begin{pmatrix} 2e^t + 8te^t \\ 6e^t - 8te^t \end{pmatrix}$$

consider the Wronskian $W(x_1, x_2)$,

$$W = \begin{vmatrix} e^t & 2e^t + 8te^t \\ -e^t & 6e^t - 8te^t \end{vmatrix}$$

$$= 6e^{2t} - \cancel{8te^{2t}} + 2e^{2t} + \cancel{8te^{2t}}$$

$$= 8e^{2t} \neq 0 \quad \forall\, t.$$

$\therefore \{x_1, x_2\}$ are linearly independent.

(ii) Check whether the vector functions $x_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$, $x_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{t}$ form a fundamental soln. set. If yes, find a fundamental matrix for ~~this~~ the system & give a general soln.

Soln: By definition, an independent solns $\{x_1, x_2\}$ is called fundamental ~~system of~~ soln. of that system.

Consider the Wronskian $W(x_1, x_2)$

$$W = \begin{vmatrix} e^{-t} & 2e^{t} \\ 2e^{-t} & 3e^{t} \end{vmatrix} = 3e^{0} - 4e^{0} = -1 \neq 0$$

$\therefore \{x_1, x_2\}$ are linearly independent. Hence $\{x_1, x_2\}$ form a fundamental ~~sy~~ soln set.

The fundamental matrix for the system is defined as

$$\begin{bmatrix} e^{-t} & 2e^{t} \\ 2e^{-t} & 3e^{t} \end{bmatrix}$$

General soln is

$$x(t) = c_1 x_1 + c_2 x_2$$

(iii) Show that $x = 2e^{2t}, y = -3e^{2t}$ and $x = e^{7t}, y = e^{7t}$ are soln of the homogeneous system

$$\frac{dx}{dt} = 5x + 2y$$
$$\frac{dy}{dt} = 3x + 4y$$

Soln: Let $x = 2e^{2t}, y = -3e^{2t}$

then $\frac{dx}{dt} = 4e^{2t}, \frac{dy}{dt} = -6e^{2t}$

our system is $\left.\begin{aligned} \frac{dx}{dt} &= 5x + 2y \\ \frac{dy}{dt} &= 3x + 4y \end{aligned}\right\}$

we can check

$4e^{2t} = 5.2e^{2t} - 2.3e^{2t}$ holds for both x & y.

& $-6e^{2t} = 3.2e^{2t} - 4.3e^{2t}$

hence $\{x = 2e^{2t}, y = -3e^{2t}\}$ is a soln. → ✓

(*) using another way let $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^{7t} \\ e^{7t} \end{pmatrix}$

our system is $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

$$\Rightarrow \begin{pmatrix} e^{7t} \\ e^{7t} \end{pmatrix}' = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} e^{7t} \\ e^{7t} \end{pmatrix} = \begin{pmatrix} 5e^{7t} + 2e^{7t} \\ 3e^{7t} + 4e^{7t} \end{pmatrix}$$

$$= \begin{pmatrix} 7e^{7t} \\ 7e^{7t} \end{pmatrix}$$ Satisfied

hence $\{x = e^{7t}, y = e^{7t}\}$ is also a soln.

→ ✓.

(5) (2) Find the general solⁿ of the homogeneous linear system with constant coefficients:

i) $\frac{dx}{dt} = 2x + 3y$

$\frac{dy}{dt} = -x - 2y$

solⁿ: given system $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

matrix $A = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$

The characteristic eqⁿ is $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & 3 \\ -1 & -2-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(-2-\lambda) + 3 = 0$

$\Rightarrow -(4-\lambda^2) + 3 = 0$

$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1.$

eigenvalues are $\lambda = 1, \lambda = -1$

eigenvector corresponding to $\lambda = 1$.

$(A - 1.I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2-1 & 3 \\ -1 & -2-1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \left.\begin{array}{l} x + 3y = 0 \\ -x - 3y = 0 \end{array}\right\}$

Scanned with CamScanner

$x = -3y.$

$\therefore$ eigenvector is $\begin{pmatrix} x \\ -\frac{1}{3}x \end{pmatrix} = x\begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix}$

eigenvector corresponding to $\lambda = -1$

$$(A + 1.I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2+1 & 3 \\ -1 & -2+1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} 3x + 3y &= 0. \\ -x - y &= 0 \end{aligned}\right\} \Rightarrow x = -y.$$

$\therefore$ eigenvector is $\begin{pmatrix} x \\ -x \end{pmatrix} = x\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

Therefore the solⁿ is

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^{t}\begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 e^{t} + c_2 e^{-t} \\ -\frac{1}{3}c_1 e^{t} - c_2 e^{-t} \end{pmatrix}$$ Ans.

(ii) $\frac{dx}{dt} = x$

$\frac{dy}{dt} = -2x + y$

Sol^n: given system $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

matrix $A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$

The characteristic eqn is $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 0 \\ -2 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)^2 = 0$

$\Rightarrow \lambda = 1, 1$ repeated root.

eigenvalue is $\lambda = 1$ with multiplicity 2.

eigenvector corresponding to $\lambda = 1$ is

$(A - I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1-1 & 0 \\ -2 & 1-1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow -2x = 0. \Rightarrow x = 0.$ & y is free ~~vector~~.

$\therefore$ eigenvector is $\begin{pmatrix} 0 \\ y \end{pmatrix} = y\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\therefore \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is ~~our~~ our first eigenvector.

33

we have only (one) eigenvector (linearly indep.).

for second vector v, we need to consider the eqn

$$(A - I)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow -2v_1 = 1.$$

$$\Rightarrow v_1 = -\tfrac{1}{2}. \text{ \& } v_2 \text{ is free.}$$

desired second vector ~~eigenvector~~ is $\begin{pmatrix} -1/2 \\ v_2 \end{pmatrix}$ & a nontrivial choice is $\begin{pmatrix} -1/2 \\ 0 \end{pmatrix}$ $\therefore$ $v = \begin{pmatrix} -1/2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} t = \begin{pmatrix} -\frac{1}{2}t \\ t \end{pmatrix}$. (by ⊛ in note). discussed earlier

And we have $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ & $\begin{pmatrix} -\frac{1}{2}t \\ t \end{pmatrix}$ are Linearly independent.

$\therefore$ a general soln of the system is

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^t \begin{pmatrix} -\frac{1}{2}t \\ t \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{c_2}{2} e^t \cdot t \\ c_1 e^t + c_2 e^t t \end{pmatrix}.$$

a general soln

$$\therefore \left.\begin{aligned} x &= -\frac{c_2}{2} t \cdot e^t \\ y &= c_1 e^t + c_2 \cdot t e^t \end{aligned}\right\} \text{Ans.}$$